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Small Corrections to Astronomic Observations

by

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Facsimile Cover

Small Corrections to Astronomic Observations

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1. The Aim

The aim is to reduce all longitudes to a common origin, the former meridian of the Greenwich transit instrument; to reduce all latitudes, longitudes and azimuths to a fixed pole; and to have all observations in terms of a single star catalogue, the FK 4.

2. Tha object of this Paper

This paper defines terms and gives formulae for the corrections to the observations. Only the minimum of explanation is given. The object is to show "How", with complete unambiguity. The reason "Why" is left for a later paper.[1]

3. Conventions

(*a*) Although at first sight inconvenient for Australia, it will be found best to stick to the conventions of the Bureau International de l'Heure (BIH) and the International Polar Motion Service:

Longitude: Positive West, Negative East of Greenwich.

Latitude: Positive North, Negative South of the Equator.

Azimuths: Clockwise from North.

(*b*) Coordinates of the pole:

x: Positive towards Greenwich
y: Positive towards 90° W., i.e., Canada

} Referred to the mean pole of epoch. Published monthly by Mt. Stromlo.

LONGITUDES

4. The Preliminary Longitude

A preliminary longitude is computed on the assumption that time signals are received at their correct nominal instants.

The star coordinates used to obtain the preliminary longitude should be corrected for *short period nutation*, and should be calculated using second differences. For details, see the Introduction to " Apparent Places of Fundamental Stars ".

When longitudes are observed by the method of Meridian Transits, the preliminary longitude should also contain the correction for *Diurnal Aberration* (Apparent Places, Table VII). See paragraph 9*a* for diurnal aberration with almucantar longitudes.

[1] See "Astronomic Longitudes in Australia", A. G. Bomford, proposed to be published in the December, 1964 issue of the Australian Surveyor.

5. Definitions

Δt = $t_m - t_o$, where t_m is the UT2 time of reception of the time signal at Mr. Stromlo, and t_o is the nominal time. Values of Δt are published monthly by Mount Stromlo in *Time Service Bulletin B.* There is an advantage if Mount Stromlo times are used in preference to corrections given by other observatories, see paragraphs 6, 7, 8.

ΔT_s = UT2 — UT1, published monthly by Mount Stromlo.

$\Delta\lambda_f$ = Correction to the mean pole of epoch, computed from the following formula using the coordinates of the field station:

$$\Delta\lambda_f = \frac{\tan\varphi}{15}(x \sin\lambda - y\cos\lambda)$$

$\Delta\lambda_{MS}$ = Correction to the mean pole at Mount Stromlo; only required prior to 1956·0 when UT2 was introduced. For Mount Stromlo, $\lambda = 140^\circ\,00'$, $\phi = -35^\circ\ 19'$.

TF: Transit time from transmitter to field station.

TM: Transit time from transmitter to Mount Stromlo. (Transit times for short wave time signals are to be calculated using Stoyko's tables (Bul. Horaire, No. 10, 1956), see page 211.) The distance WWVH to Mount Stromlo is 8,559 kms., and the accepted delay 30·8 milliseconds.

PE: Personal equation of the observer. Positive if he observes too early, negative if he observes too late, as is more usual.

Lag: Lag in the audio filter, positive if the relay acts late, as is usual.

6. Catalogues

At 1962·0, the FK4 catalogue came into universal use, and for 1962 and 1963, "Apparent Places" must be corrected accordingly. From 1964·0, "Apparent Places" is on the FK4, and no corrections are necessary.

Prior to 1961·0, both Mount Stromlo and surveyors were using the FK3. In so far as both may be considered to be observing the same stars, errors in the catalogue cancel (provided that Mount Stromlo time signal corrections have been used), and no corrections need be made.

During 1961, Mount Stromlo was using a PZT catalogue which was in sympathy with the FK4. Surveyors meanwhile continued to use the FK3. A correction is therefore necessary. The FK3-FK4 corrections vary only very slowly with time, and for 1961 observations it is therefore best for surveyors to correct Right Ascensions using the FK4 conversion tables for 1962, tables for 1961 not being available. If longitudes have already been calculated using FK3 RA's, recomputation is not required. The preliminary FK3 longitude can simply be corrected by the mean FK4 correction to the RA's:

$$\frac{-\Sigma\, n\, F}{N}$$

where F is the FK4 correction to a star, n the number of times that star was observed, and N is the total number of observations.

7. The former Meridian of Greenwich

Longitude 0° is traditionally defined by the axis of the transit instrument at Greenwich observatory, and all the observatories of the BIH have a conventional longitude which is used to convert their local times to Greenwich time.

In the early 1950's, observations at Greenwich ceased. Observatories continued to use the same conventional longitudes until 1962·0. During this decade, great advances were made in the accuracy of time determinations, notably in the elimination of personal equation. By 1961, it was clear that the old conventional longitudes were in error. New conventional longitudes were introduced at 1962·0, to bring the zero of longitude back on to the former meridian of Greenwich as closely as possible.

The change in Mount Stromlo's conventional longitude was $0^s.056$. However, the inaccuracy of Mount Stromlo's time determination, due to personal equations, etc., is also known to some extent, and the correction to be applied to bring Mount Stromlo longitudes on to the former meridian of Greenwich varies from year to year, as follows:

	milliseconds
1962 and later:	No correction
1961	—56
1960	—56
1959	—55
1958	—57
1957	—92
1956	—85
1955	—82
1954	—67
1953	—81
1952	—86
1951	—85
1950	—85

The value for the year should be applied without interpolation.

8. Time Signal Corrections from observatories other than Mt. Stromlo

In paragraphs 6 and 7, the assumption is made that the corrections to time signals have been obtained from observations at Mount Stromlo. If this is not the case, the corrections will be different. It would seem simplest to remove any time signal correction obtained from Perth, Washington, etc., and apply the Mount Stromlo corrections, and then proceed as in paragraphs 6 and 7. Paragraph 10 assumes that this has been done.

9. Almucantar Longitudes

(*a*) *Diurnal Aberration:*

It is better to make no correction to the individual Right Ascensions, but to apply a single correction to the Preliminary longitude of

$$-0^s.0213 \sin E$$

where E is the elevation of the almucantar.

(*b*) *Personal Equation:*

In 1962, numerous Almucantar Longitudes were observed with Wild T3's in the Woomera area. At 12 stations, longitudes had previously been determined with a T4, by various observers in different years. Four different T3 observers made observations at various T4 stations, and a total of 19 comparisons is available. On average, the T3 longitudes required a correction of —1″·32 to bring them into sympathy with the T4 observations. No very great accuracy can be claimed for this figure; but personal equation is generally considered to be of this order, and in the absence of any other test data, it is proposed that a correction of

$$-1''.32 \text{ or } -0^s.088$$

shall be applied to all Almucantar Longitudes observed with a fixed wire graticule, unless the observer's personal equation has been otherwise determined.

10. Eccentricity

The eccentricity of the observation station is best reduced to so many feet west and north of the trig station, if the two are not coincident. The easting can then be converted into seconds of arc of longitude, using Table IV of the *Text Book of Topographical Surveying*, for example (or Table III for latitudes). Tables for any spheroid suffice. It is then wise to convert the correction from arc to time, and to apply it in that form, as in paragraph 11. Otherwise there is a danger that at a later date someone may want a value for the longitude in time, and take the value which has not had the eccentric correction applied. If the eccentric is west or north of the trig, the correction is negative.

11. The FK4 Greenwich Longitude

The astronomic longitude, to be known as " FK4-Greenwich ", is then given, working in time throughout, by:

Astronomic longitude = Preliminary Longitude, positive west, in time.
+ Δt
— $\Delta\lambda_f$ (+ $\Delta\lambda_{MS}$, for Mt. Stromlo, prior to 1956·0 only)
— ΔT_s
+ TF
— TM
+ PE (usually Almucantar observations only)
+ Lag (if known)
+ Diurnal Aberration (Almucantar only, always negative)
+ Correction to Greenwich (prior to 1962·0 only)
+ Eccentricity correction, in time.

LATITUDES

12. Preliminary Latitude

As with longitudes, it is customary to compute a preliminary value for the latitude first, to which small corrections are applied. In computing the preliminary latitude, the declinations should be corrected as follows:

(*a*) *If using the FK*3:

Short period nutation
Second differences } see Introduction to *Apparent Places.*

FK3-FK4, using the 1962 tables for all years prior to 1962·0.

(*b*) *If using the Boss catalogue:*

When available (probably 1963), use tables converting declinations from Boss to FK4.

Currently, convert Boss declinations first to FK3, using *Tafeln zur Reduktion des Systems des General Catalogue auf das System FK*3 by A. Kopff, published in Berlin in 1940. Secondly, convert from FK3 to FK4 by using tables D41, supplied by the Astronomische Rechen-Institut, photo copies of which are available from Division of National Mapping.

Corrections for short-period terms of nutation are (starting with 1960) included in the Besselian Day Numbers, and no further correction is necessary.

13. Diurnal Abberration

No correction is required, since altitudes are measured in the meridian.

14. Movement of the Pole

The correction to the mean pole of epoch is

$$\Delta\varphi_P = -(x \cos\lambda + y \sin\lambda)$$

The mean pole of epoch is not fixed, and it might be thought better to apply an additional correction to, say, the mean pole of 1900-05. The movement of the mean pole of epoch in a year or even a decade is however trivial, and the effect of omitting the correction is to measure latitudes from the mean pole of 1958, or whatever the average date of the latitude observations may be. If it subsequently thought better to revert to the pole of 1900-05, a block correction is easily made. All longitudes are necessarily reduced to the mean pole of epoch, which is used by the Bureau International de l'Heure and by all time observatories.

15. Height above Sea Level

It is customary to apply the following estimate for the tilt between the equipotential surface at the observation station and the geoid at sea level:

$$\Delta\varphi_H = -52'' \times 10^{-6} \times h \sin 2\varphi$$

where h is in feet above sea level. In the southern hemisphere, φ and sin 2φ are negative, $\Delta\varphi_H$ becomes positive, and in both hemispheres the effect is thus to reduce the numerical value of the latitude. This correction is not applicable for all purposes, see paragraph 25 (*c*).

16. Catalogues

If the preliminary latitude has been computed with declinations which have not been reduced to the FK4, recomputation is not required, but a correction should be applied.

The correction to the latitude is the mean of the corrections to the declinations, with the same sign, for all upper transit stars both north or south of the zenith. However, if any stars have been observed at lower transit, the signs of the corrections to their declinations must be reversed before inclusion in the mean.

The labour of bringing all latitudes on to the FK4 system is thus relatively slight, and the work can be undertaken by relatively unskilled labour.

17. The FK4 Latitude

The astronomic latitude, to be known as the FK4 latitude, is thus given by:

Astronomic Latitude = Preliminary Latitude (positive north)
+ $\Delta\varphi_P$
+ $\Delta\varphi_H$
+ Correction to catalogue if the Preliminary Latitude is not in terms of FK4
+ Eccentricity correction if required (see paragraph 10)

AZIMUTHS BY SIGMA OCTANTIS

18. Definitions

The use of the following terms is recommended:

(*a*) *Geodetic Azimuth*: The azimuth brought forward from a distant point by ground survey and geodetic computation.

(*b*) *Astronomic Azimuth*: The azimuth found by observing the stars, independent of any geodetic spheroid or datum. It is the Preliminary Azimuth (paragraph 19) corrected as in paragraphs 20-24; but without the correction for the Laplace Equation, or the correction for the Deviation of the Vertical. Once determined, the astronomic azimuth is constant, and can be only be altered by re-observation.

(*c*) *Laplace Azimuth*: The azimuth obtained by correcting the Astronomic Azimuth for Laplace's Equation and (when not negligible) for the Deviation of the Vertical. The Laplace Azimuth is used to control the Geodetic Azimuth. It changes whenever the geodetic latitude and lontigude are changed, whether this is due to a new survey, a new spheroid or a new origin.

19. The Preliminary Azimuth

(*a*) The mean preliminary azimuth should be computed using astronomic values of latitude and longitude. Prior to 1962·0, the RA and Dec should be taken from the FK3; after 1962·0 from the FK4. Astronomic coordinates are given at Upper Transit at Greenwich, not at 0 hrs. UT, and care is necessary to see that the interpolation is done correctly.

(*b*) Certain forms for hand computation in common use assume that

$$\tan A = A'' \times \tan 1''$$

This approximation can lead to errors of over 0·5″ when Sigma Octantis is at elongation, and it should not be used.

(*c*) If the observations on Face Left and Face Right are separated by less than 2 minutes of time, the *curvature correction* will nearly always be less than 0″.04 and may be ignored. Otherwise, the curvature correction, as shown in the Table, should be applied to each zero before the preliminary azimuth is meaned.

TABLE

Curvature Correction for Sigma Octantis

Curvature Correction

t = time interval in minutes between pointings on FL and FR.

A = Computed azimuth of star.

Correction is in seconds of arc and is to be applied to the final azimuth of the RO, with sign as follows:

Azimuth of star: 179° 180° 181°

Sign of correction: + | —

A'	1^m	2^m	3^m	4^m	5^m	6^m	7^m	8^m
0° 00′	·00	·00	·00	·00	·00	·00	·00	·00
10′	·00	·01	·01	·02	·04	·05	·07	·09
20′	·00	·01	·03	·05	·07	·10	·14	·18
30′	·00	·02	·04	·07	·11	·15	·21	·27
40′	·01	·02	·05	·09	·14	·20	·28	·36
50′	·01	·03	·06	·11	·18	·26	·35	·46
1° 00′	·01	·03	·08	·14	·21	·31	·42	·55
10′	·01	·04	·09	·16	·25	·36	·49	·64
20′	·01	·05	·10	·18	·29	·41	·56	·73
30′	·01	·05	·12	·21	·32	·46	·63	·82
40′	·01	·06	·13	·23	·36	·51	·70	·91
50′	·02	·06	·14	·25	·39	·57	·77	1·01
2° 00′	0·2	0·7	·15	·27	·43	·62	·85	1·10

Geodesy (1962), p.330, formula (5.53).

(*d*) *The stride level correction*: If when the telescope is pointed to the star, the left or east end of the bubble is higher than the right or west end, the angle read on the horizontal circle of the theodolite is too too small, and needs to be increased.

If the stride level is numbered outwards from the centre, the correction is

$$+\frac{d}{n}(\Sigma L - \Sigma R)\tan E$$

where d is the value in seconds of 1 division of the bubble

n is the number of bubble readings

ΣL is the sum of the readings at the left end

ΣR is the sum of the readings at the right end, and

E is the elevation of the star.

If the stride level is numbered continuously from one end, let $\Sigma(L + R)_W$ indicate the sum of the readings at both ends of the bubble when the zero end is in the west, and $\Sigma(L + R)_E$ be the sum of both the bubble readings when the zero end is in the east. Then the correction is

$$+\frac{d}{n}[\Sigma(L + R)_W - \Sigma(L + R)_E]\tan E$$

Note that these stride corrections are applicable to the horizontal angle to the star as read on the theodolite, not to the computed azimuth of the star, nor to the azimuth of the RO.

It may sometimes happen that the elevation of the RO is not negligible, in which case the stride level should be read and an identical correction should be applied to the circle reading to the RO. If the ray is a depression, E and $\tan E$ change sign.

(*e*) If computations were made using a geodetic longitude which is subsequently found to be significantly different from the astronomical longitude, complete recomputation can be avoided. A correction graph of $\Delta A''$ against $\Delta\lambda''$ can be drawn from the following approximate formula, and each zero can then be individually corrected:

$$\Delta A'' = - P'' \times \sec E \times \cos t.\ \Delta\lambda''.\ \sin 1''.$$

where P'' is the polar distance of the star in seconds. However, with electronic computations, it is simpler to put the revised data through the computer a second time.

20. Eccentric Observations

The preliminary azimuth should be corrected by the usual formulae if either the theodolite (or the light) was not over the trig point (or the RO). If the theodolite was eccentric, it is in addition necessary to correct for the convergence of the meridians between the eccentric and the trig.

The numerical value of the correction is:—

(Difference in longitude in seconds of arc between eccentric and trig) × (sin Lat.)

In the southern hemisphere, if the eccentric is east of the trig, the eccentric azimuth needs to be increased. If the eccentric is west of the trig, the eccentric azimuth needs to be decreased. These rules should be reversed in the northern hemisphere.

21. Movement of the Pole

The correction to be applied to the preliminary azimuth is

$$\Delta A_P = (x \sin \lambda - y \cos \lambda) \sec \phi$$

22. Diurnal Aberration

For Sigma Octantis, the preliminary azimuth should be corrected by $-0''\cdot32$. For stars further from the pole, the general formula is

$$+ 0''\cdot32 \cos A \cos \phi \sec E$$

The sign of the correction varies with cos A.

23. Skew-Normals, or Elevation of the RO

In mountainous country, the skew-normal correction becomes significant and should be applied to all horizontal directions, including an astronomic RO. The numerical value of the correction is

$$33'' \times 10^{-6} \times h_2\ x \sin 2A \cos^2 \phi$$

where h_2 is the height in feet above sea level of the RO (*not* the height of the observing instrument).

The sign of the correction to the preliminary azimuth is positive if the RO is to the NE or SW, and negative if the RO is SE or NW. The signs are the same in both hemispheres.

24. The Astronomic Azimuth

The astronomic azimuth is then given by:

Astronomic Azimuth = Preliminary azimuth
+ Eccentricity corrections
+ Convergence
+ Pole
+ Diurnal aberration
+ Skew normals

LAPLACE'S EQUATION AND DEVIATION OF THE VERTICAL

25. The Laplace Azimuth

(*a*) The corrections in this section depend on the geodetic latitude and longitude, and may thus vary arbitrarily from time to time. The spheroid and origin used must always be clearly stated.

(*b*) As the Clarke 1858 spheroid (and its numerous origins) is rapidly becoming obsolete, it is recommended that on Laplace Station summary forms all entries dependent on the geodetic latitude and longitude should be left blank until the National Geodetic Adjustment is complete.

(*c*) *Deviation of the Vertical.* To the astronomic azimuth add:

$$- \zeta \tan E$$

where ζ is the component of the deviation of the vertical at right angles to the line (for details, see paragraph 26), and E is the observed angular elevation of the RO. Ideally, E should be observed at the time of the azimuth observations, and should not be corrected for refraction, nor for heights of instruments and beacons. If the ray is a depression, E will be negative. The sign is the same in both hemispheres.

In computing ζ for this purpose, the astronomical latitude should not, strictly, have been reduced to sea level by $\Delta\phi_H$ as in paragraph 15. However, the difference will generally be negligible.

In mountainous country, this correction can very easily attain a value of 1", and it should be applied to all horizontal angles (not only astronomical ROs) whenever E exceeds say 20′, and the deviation of the vertical is known.

(*d*) *The Laplace Correction.* To the Astronomic Azimuth add:

$$\text{(Astronomic Longitude — Geodetic Longitude)} \times \sin\phi$$

where longitudes are positive west and latitudes positive north. In Australia and New Guinea, the correction to the astronomic azimuth is positive if the numerical value of the astronomic longitude exceeds that of the geodetic longitude.

(*e*) *Reciprocal azimuths.* When astronomic observations have been made at both ends of a line, the following procedure is recommended:

(i) Compute the Laplace azimuth at each end using the best geodetic coordinates on the best spheroid currently available.

(ii) Obtain the difference, ΔA, between the forward and reverse geodetic azimuth of the line from Clarke's formula or from

$$\Delta A = \Delta\varphi_G \sin\phi_m$$

where $\Delta\varphi_G$ is the difference between the geodetic longitudes and ϕ_m is the mean geodetic latitude.

(iii) Apply ΔA to the Laplace azimuth at one end of the line and mean with the Laplace azimuth at the other end, to obtain the *Mean Laplace Azimuth.*

(iv) Remove ΔA from one end, and the Laplace corrections (and if applied, the deviation correction) from both ends, and quote the resulting *Mean Reciprocal Astronomic Azimuth* on the respective Laplace station summaries, stating in the remarks column the values of the Laplace corrections, Deviation corrections and of ΔA used.

DEVIATIONS OF THE VERTICAL

26. Definitions

The deviation of the vertical is always resolved into two components. Usually these are the components in the meridian and in the prime vertical.

Xi = ξ = component of deviation in the meridian, positive when geoid zenith is north of spheroid normal.
= astronomic latitude — geodetic latitude.

Eta = η = component of deviation in the prime vertical, positive when geoid zenith is west of spheroidal normal.
= (astronomic longitude — geodetic longitude) cos ϕ
= — (astronomic azimuth — geodetic azimuth) cot ϕ

27. Components relative to a line in Azimuth A

For geoid studies, and for the correction of azimuths (see paragraph 25*c*) we need to consider the components of the deviation of the vertical along, and at right angles to, a line in a specified azimuth A.

Chi $= \chi =$ component of deviation in azimuth A, positive when the geoid zenith is behind the spheroidal normal.

$= -\xi \cos A + \eta \sin A$

Zeta $= \zeta =$ component of deviation at right angles to azimuth A, positive when the geoidal zenith is to the left of the spheroidal normal.

$= + \xi \sin A + \eta \cos A.$

28. Rise of the geoid above the spheroid

If χ is known at two points L units apart, and N is the height of the geoid above the spheroid,

$$N_b - N_a = + \tfrac{1}{2}(\chi''_b + \chi''_a).\ L.\ \sin 1'',$$

in the same units as L. χ_a and χ_b must both be computed in the direction A to B.

29. Vertical angles referred to the spheroid

If E is a vertical angle observed with reference to the geoid (as is always the case), and E' is the vertical angle required above the spheroid, then

$$E' = E + \chi$$

E and E' are considered positive if the angle is an elevation, negative if a depression. (χ is defined in paragraph 27).

Appendix "A"

THE JJY TIME SIGNAL

Frequently in Australia, JJY and WWVH time signals are received simultaneously. On precise work, a time signal receiver should be used which filters out JJY leaving WWVH, and there is then no difficulty in applying the appropriate corrections. However, such receivers are not always available when observations are made with geodetic theodolites.

(2) The personal equation of — 0ˢ·088 mentioned in paragraph 9 (*b*) was obtained by applying corrections for WWVH to a mixed JJY/WWVH signal. The personal equation of the observer must whenever possible be determined by calibration at an impersonal longitude station; but if this is not being done, and — 0ˢ·088 is being used for want of anything better, then corrections for WWVH should be used even if JJY was also heard.

(3) If the observer is determining his own personal equation by calibrating at a T4 station, it make little difference whether he uses WWVH or JJY corrections provided he uses the same at both his field and calibration stations. As JJY is now infrequently monitored by Mount Stromlo, it will generally be convenient to apply corrections for WWVH.

(4) Nevertheless, the following points should be borne in mind:

(*a*) While the WWVH signal consists of a toc 5 ms long, the start of which marks the precise second, the JJY signal consists of a toc 20 ms, the *end* of which marks the precise second.

(*b*) While observers are trained to press the tappet at the start of a long toc, it is believed that for short tocs of 20 ms. or less, they tend to press the tappet at the " centre of gravity " of the signal.

(*c*) If the field station is equidistant from Tokyo and Hawaii (which is approximately the case over much of Australia) the observer will hear a combined toc of 25 ms, and the exact second will be 20 ms after the start. We may assume the observer will press this tappet say 10 ms before the exact second.

(*d*) Therefore, if JJY signals are being received, it is suggested that the values of Δt taken from the time service bulletins should be altered by — 0·010 seconds, and TF and TM should be calculated for Tokyo.

(*e*) If Δt for JJY is not given in the bulletin, we may note that the times of emission of JJY and WWVH are co-ordinated within 5 ms and generally much less, and can therefore proceed as follows:

Use Δt for WWVH but change it by — 0·010 seconds, as above.

Use TM for Hawaii-Stromlo, 31 ms.

Use TF for the distance from Tokyo to the field station.

(5) But in practice, it is suggested that the WWVH corrections should be used, even if JJY is head, provided that:

(*a*) On the best impersonal work, a time signal receiver is used which filters out JJY.

(*b*) On lesser work, the observer either determines his own personal equation by calibration at an impersonal longitude station, or is using the figure of — 0^s·088 mentioned in paragraph 9 (*b*).

Appendix "B"

ESTIMATES OF ACCURACY

In astronomical work, particularly azimuths and longitudes, any estimates of probable errors obtained by multiplying the standard deviation of the observations by 0·6745 can easily be an error by a factor of ten, due to errors arising from atmospheric refraction and personal equation. No probable errors are therefore quoted on the new Laplace station summaries.

(2) Instead, the standard deviations of the observations are entered, and the standard errors of their means. To avoid any ambiguity, the formulae defining these terms are printed on the forms.

(3) The rigorous calculation of the standard deviations can be a tedious task. If several stations have been observed by the same man with the same instrument within a reasonably short period—say a month or two—it is satisfactory to estimate standard deviations from the *mean range* of sets of observations, using the following labour-saving formulae:

Standard deviation of n pairs = (Mean range of n pairs) $\times$ $1/d$

Standard Error of mean of n pairs = (Mean range of n pairs) $\times$ $1/d\sqrt{n}$

Tables of $1/d$ and $1/d\sqrt{n}$ for various values of n are given below.

(4) A standard deviation estimated in this way from a range of a single set is worthless. For the estimate to be realistic, the range used in the formulae must be the mean range of at least 4 sets and preferably many more. It is therefore best to calculate a figure from the mean range of all the sets observed by the same observer and instrument at several stations on one journey and to enter this figure on all the relevant summaries.

TABLE

n	$1/d$	$1/d\sqrt{n}$
2	0·886	0·627
3	0·591	0·341
4	0·486	0·243
5	0·430	0·192
6	0·395	0·161
7	0·370	0·140
8	0·351	0·124
9	0·337	0·112
10	0·325	0·103

See Moroney, 1951, *Facts from Figures*, p. 155.

G. Bomford, 1962, *Geodesy*, Second Edition, p. 510.

Olliver, January 1963, *Survey Review*, p. 51.

TABLE

APPARENT VELOCITY OF PROPAGATION OF SHORT WAVE RADIO TIME SIGNALS

Anna Stoyko, Bulletin Horaire, No. 10, 1956

Distance (*kms*).	*Velocity* (*km./sec.*)	*Delay* (*milliseconds*)
1,000	254 300	3·9
1,500	258 300	5·8
2,000	261 500	7·6
2,500	264 200	9·5
3,000	266 400	11·3
3,500	268 200	13·1
4,000	269 800	14·8
4,500	271 200	16·6
5,000	272 400	18·4
6,000	274 300	21·9
7,000	275 900	25·4
8,000	277 200	28·9
9,000	278 300	32·3
10,000	279 200	35·8
11,000	280 000	39·3
12,000	280 600	42·8
13,000	281 200	46·2
14,000	281 800	49·7
15,000	282 200	53·2
16,000	282 600	56·6
17,000	283 000	60·1
18,000	283 300	63·5
19,000	283 600	67·0
20,000	283 900	70·4
21,000	284 200	73·9
22,000	284 400	77·4
23,000	284 600	80·8
24,000	284 800	84·3
25,000	285 000	87·7
26,000	285 200	91·2
27,000	285 300	94·6
28,000	285 500	98·1
29,000	285 600	101·5
30,000	285 800	105·0
31,000	285 900	108·4
32,000	286 000	111·9
33,000	286 100	115·3
34,000	286 200	118·8
35,000	286 300	122·2
36,000	286 400	125·7
37,000	286 500	129·1
38,000	286 600	132·6
39,000	286 700	136·0
40,000	286 800	139·5